Problem 1

$$\omega_r = \frac{2}{P}\omega_e = \frac{2}{4}(2\pi \times 60) = 188.4956 \text{ r/sec}$$

$$\omega_{m,r} = (1-\sigma_r)\omega_r = (1-0.1477)(188.4956) = 160.6548 \text{ r/sec.}$$

$$\omega_{m_1} = 0.45\,\omega_{m,r} = 72.2946 \text{ rad/sec}$$

$$\omega_{m_2} = 0.80\,\omega_{m,r} = 128.5238 \text{ rad/sec}$$

$$\sigma_1 = \frac{\omega_r - \omega_{m_1}}{\omega_r} = 0.6165$$

$$\sigma_2 = \frac{\omega_r - \omega_{m_2}}{\omega_r} = 0.3182$$

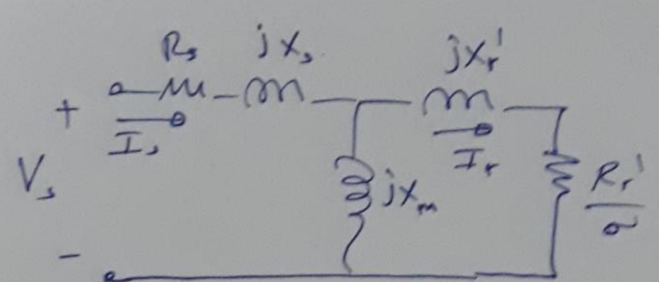
$$T_{L_1} = T_{f+w,1} + C\omega_{m_1}^2 \quad , \quad T_{L_2} = T_{f+w,2} + C\omega_{m_2}^2$$

$$T_{L_1} = 0.01\omega_{m_1} + (0.0005+C)\omega_{m_1}^2 = 144.4521 \text{ N.m}$$

$$T_{L_2} = 0.01\omega_{m_2} + (0.0005+C)\omega_{m_2}^2 = 455.5404 \text{ N.m}$$

$$T_{L_1} = \frac{3 I_{r_1}'^{\,2} R_r'}{\omega_r \sigma_1} \Rightarrow I_{r_1}' = \sqrt{\frac{T_{L_1}\omega_r \sigma_1}{3R_r'}} = 159.48 \text{ A}$$

$$T_{L_2} = \frac{3 I_{r_2}'^{\,2} R_r'}{\omega_r \sigma_2} \Rightarrow I_{r_2} = \sqrt{\frac{T_{L_2}\omega_r \sigma_2}{3R_r'}} = 203.45 \text{ A}.$$



$$I_r = \frac{jX_m}{(jX_m) + \left(\frac{R_r'}{\sigma} + jX_r'\right)} I_s$$

$$I_s = \left(\frac{jX_m + \frac{R_r'}{\sigma} + jX_r'}{jX_m}\right) I_r$$

current divider

$$I_{s_1} = \frac{\frac{R_r'}{\sigma_1} + j(X_m + X_r')}{jX_m} \cdot I_{r_1} = 161.49 \angle -2.02^\circ \text{ A}.$$

$$I_{s2} = \frac{\frac{R_r'}{\sigma_2} + j(X_m + X_r')}{jX_m} \cdot I_{r_2} = 206.02 \angle -2.02^\circ \text{ A}.$$

$$V_{s_1} = (R_s + jX_s) I_{s_1} + \left(\frac{R_r'}{\sigma_1} + jX_r'\right) I_{r_1} = 71.53 \angle 29.39^\circ \ \underline{V}$$

$$V_{s_2} = (R_s + jX_s) I_{s_2} + \left(\frac{R_r'}{\sigma_2} + jX_r'\right) I_{r_2} = 154.23 \angle 16.87^\circ \ \underline{V}$$

Problem 2

1) $$T_{el,max} = \frac{V^2}{2\omega_f \left[R_s \mp \sqrt{R_s^2 + (X_s + X_r')^2} \right]}$$ +: motoring
−: braking

$$= \frac{V^2}{2\omega_f^2} \cdot \frac{1}{\left[R_s \mp \sqrt{R_s^2 + (X_s + X_r')^2} \right]} \cdot \omega_f \qquad V/\omega_f = \text{constant}$$

$$\gamma = \frac{T_{el,max}(f = 12Hz)}{T_{el,max}(f = 60Hz)} = \underbrace{\frac{R_s \mp \sqrt{R_s^2 + (2\pi \times 60)^2 (L_s + L_r')^2}}{R_s \mp \sqrt{R_s^2 + (2\pi \times 12)^2 (L_s + L_r')^2}}}_{} * \frac{12}{60}$$

$R_s = 0.074\ \Omega$
$L_s = L_r' = \frac{0.12}{2\pi \times 60}$

For motoring: $\gamma = \boxed{0.68}$

For braking: $\gamma = \boxed{1.46}$

2) At starting $\sigma = 1$

$$T_{el} = \frac{V^2}{\omega_f \left[(R_r' + R_s)^2 + (X_r' + X_s)^2 \right]} = \frac{V^2}{\omega_f^2} \cdot \frac{1}{(R_s + R_r')^2 + (X_r' + X_s)^2} \cdot \omega_f$$

$$\frac{T_{el}(f = 12)}{T_{el}(f = 60)} = \frac{(R_s + R_r')^2 + (X_r' + X_s)^2 \big|_{f=60}}{(R_s + R_r')^2 + (X_r' + X_s)^2 \big|_{f=12}} \times \frac{12}{60} = \boxed{2.6}$$

$$I_r' = \frac{V / \sqrt{3}}{\sqrt{R_r'^2 + X_r'^2}}$$

$$\frac{I_r'(f = 12)}{I_r'(f = 60)} = \frac{V(f = 12)}{V(f = 60)} \times \sqrt{\frac{R_r'^2 + (2\pi \times 60)^2 L_r'^2}{R_r'^2 + (2\pi \times 12)^2 L_r'^2}} = \boxed{0.72}$$

problem 3

1) $T_{el,r} = \dfrac{3 I_{r,r}'^{2} R_r'}{\sigma \omega_s}$

$\underline{\sigma_r} = \dfrac{n_s - n_m}{n_s}$

$n_s = \dfrac{120}{P} f_e = \dfrac{120}{6}(50) = 1000 \text{ rpm}$

$\sigma_r = \dfrac{1000 - 960}{1000} = 0.04$

$\underline{\omega_s} = 1000\left(\dfrac{\pi}{30}\right) = 104.7 \text{ rad/sec}$

$I_{r,r}' = \left|\dfrac{Z_m}{Z_m + Z_r'}\right| I_{s,r}$

$I_{s,r} = \dfrac{400/\sqrt{3}}{|Z_{in}|}$

$Z_{in} = Z_s + \dfrac{Z_r' Z_m}{Z_r' + Z_m}$

$Z_s = R_s + jX_s = 1.55\angle 75^\circ \ \Omega$

$Z_r' = \dfrac{R_r'}{\sigma} + jX_r' = 5.22\angle 16.7^\circ \ \Omega$

$Z_m = j30 \ \Omega$

$\Rightarrow Z_{in} = 6\angle 37^\circ \ \Omega$

$I_{s,r} = \dfrac{400/\sqrt{3}}{6} = 38.5 A$

$\underline{I_{r,r}'} = 36.22 A \Rightarrow E_r = |Z_r'| I_{r,r}' = 189 V$

$\Rightarrow T_{el,r} = \dfrac{3(36.22)^2(0.2)}{0.04(104.7)} = 188 \text{ N.m}$

Now, $T_{ex} = 3P\lambda_m^2 \dfrac{R_r'/(p\omega_\sigma)}{(R_r'/p\omega_\sigma)^2 + L_r'^2}$

$T_{ex} = \dfrac{188}{2}$

$P = 6$

$R_r' = 0.2\,\Omega$

$L_r' = \dfrac{1.5}{2\pi(50)}\,H$

$\lambda_m = L_m I_m$, $I_m = \dfrac{E_r}{\omega_e L_m} \Rightarrow \lambda_m = \dfrac{E_r}{\omega_e} = \dfrac{189}{2\pi(50)} = 0.63\ H.A$

$\Rightarrow$ solve for $\sigma \Rightarrow \sigma = 0.0374$

~~At~~ $I_s = I_r' + I_m$

$I_r' = \dfrac{E}{Z_r'}$, $I_m = \dfrac{E}{jX_m}$

taking E as a reference

At 25 Hz $\Rightarrow E = 0.5 \times 189 = 94.5\,\underline{V}$

$Z_r' = \dfrac{R_r'}{\sigma} + jX_r'(25Hz) = 5.4\angle 8^\circ\ \Omega$

$\Rightarrow I_r' = \dfrac{94.5}{5.4\angle 8^\circ} = 17.5\angle -8^\circ\ A$

$I_m = \dfrac{E}{jX_m(25Hz)} = \dfrac{94.5}{j(0.5\times 30)} = 6.3\angle -90^\circ\ A$

$I_s = 17.5\angle -8^\circ + 6.3\angle -90^\circ$

$\Rightarrow \boxed{I_s = 19.85\ A}$.

2) slip speed in rpm at $T_{el,r}$ and $f_e = 50 Hz$

$n_\sigma = \sigma n_f = 0.04(1000) = 40$ rpm

Since the torque-speed curve is straight line, n_σ at half $T_{el,r}$ is $n_{\sigma_2} = 0.5 * 40 = 20$ rpm

At 25 Hz $\Rightarrow n_f = \frac{25}{50}(500) = 500$ rpm

$$\sigma_2 = \frac{500 - 480}{500} = 0.04$$

$I_s = I_m + I_r'$

$I_r' = \frac{E}{Z_r'}$, $I_m = \frac{E}{jX_m}$

taking E as a reference

$Z_r' = \frac{R_r'}{\sigma} + jX_r'(25 Hz) = 5.06 \angle 8.5^\circ$ A

$$I_r' = \frac{94.5}{5.06 \angle 8.5^\circ} = 18.7 \angle -8.5^\circ \text{ A}$$

$$I_m = \frac{94.5}{j(0.5(30))} = 6.3 \angle -90^\circ \text{ A}$$

$I_s = 18.7 \angle -8.5^\circ + 6.3 \angle -90^\circ$

$\boxed{I_s = 20.6 \text{ A}}$

3) $n_\sigma = -40$ rpm

$n_f = n_m + n_\sigma = 800 - 40 = 760$ rpm

$n_f = \frac{120}{6} f_e \Rightarrow \boxed{f_e = 38\ Hz}$.

At 38 Hz $\Rightarrow E = \frac{38}{50} \times 189 = 143.64\ \underline{V}$

$\sigma = \frac{-40}{760} = -0.0526$

$Z_r' = \frac{R_r'}{\sigma} + jX_r'(38Hz) = 3.97\angle 163.3^\circ\ \Omega$

Taking E as a reference

$I_r' = \frac{E}{Z_r'} = 36.2\angle -163.3^\circ\ A$

$I_s = 36.2\angle -163.3^\circ + 6.3\angle -90^\circ$

$\boxed{I_s = 38.52\angle -154^\circ\ A}$.

$V = Z_s I_s + E = \left(0.4 + j1.5 \times \frac{38}{50}\right)(38.52\angle -154^\circ) + 143.64$

$\boxed{V = 156\angle -17.3^\circ\ \underline{V}}$

Problem 4

$n_s = \frac{120}{P} f_e = \frac{120}{6} \times 60 = 1200 \text{ rpm}$

$\omega_s = 1200 \left(\frac{\pi}{30}\right) = 125.66 \text{ rad/sec}$

$\sigma = \frac{1200 - 1164}{1200} = 0.03$

Without rotor resistance control,

$$T_{ex} = \frac{V^2 R_r'/\sigma}{\omega_s\left[\left(R_s + \frac{R_r'}{\sigma}\right)^2 + \left(X_s + X_r'\right)^2\right]} = 78.5 \text{ N.m}$$

1.) at the breakdown torque

$$\frac{R'_{r,new}}{\sigma} = \sqrt{R_s^2 + (X_s + X_r')^2}$$

when the breakdown occurs at standstill

$$R'_{r,new} = \sqrt{R_s^2 + (X_s + X_r')^2}$$

$$= \sqrt{(0.4)^2 + (1.8 + 1.8)^2} = 3.6\ \Omega$$

$$R'_{r,new} = R_r' + R_{eq}^{*\prime} + R'_{f,eq}$$

$R'_{f,eq} + R_{eq}^{*\prime} = 3.6 - 0.6 = 3\ \Omega$

$R_{f,eq} + R_{eq}^{*} = \frac{3}{a_T^2} = 0.48\ \Omega$

$R_{f,eq} = 0.5 R_f = 0.5(0.02) = 0.01\ \Omega$

$\Rightarrow R_{eq}^{*} = 0.47\ \Omega \Rightarrow R_{eq}^{*} = 0.5(1-\overset{0}{\beta}) R \Rightarrow \boxed{R = 0.94\ \Omega}$

2.) $$T_{ee} = \frac{V^2 R'_{r,new}/\sigma}{\omega_r\left[\left(\frac{R'_{r,new}}{\sigma}+R_s\right)^2+(X'_r+X_s)^2\right]} = 1.5 \times 78.5$$

$$\sigma = \frac{1200-960}{1200} = 0.2$$

solve for $R'_{r,new}$

$\Rightarrow R'_{r,new} \approx 2.3\ \Omega$

$R'_r + R'_{f,eq} + R^{*\prime}_{eq} = 2.3\ \Omega$

$R'_{f,eq} + R^{*\prime}_{eq} = 1.7\ \Omega$

$R_{f,eq} + R^{*}_{eq} = 1.7/a_T^2 = 0.272\ \Omega$

$0.01 + 0.5(1-\delta)R = 0.272$

$0.01 + 0.5(1-\delta)(0.94) = 0.272 \Rightarrow \delta = 0.4$

3.) $R'_{r,new} = R'_r + R'_{f,eq} + R^{*\prime}_{eq}$

$$= 0.6 + a_T^2\left(\underbrace{R_{f,eq}}_{0.5R_f} + 0.5(1-\delta)R\right)$$

$$= 0.6 + (2.5)^2\left[0.5(0.02) + 0.5(0.4)(0.94)\right]$$

$$= 1.84\ \Omega$$

$$1.5 \times 78.5 = \frac{(400)^2\, 1.84/\sigma}{125.66\left[\left(\frac{1.84}{\sigma}+0.4\right)^2+(3.6)^2\right]}$$

solve for $\sigma \Rightarrow \sigma \approx 0.15$

$n_m = (1-\sigma)n_s = (1-0.15)(1200) = 1020$ rpm

problem 5

1) $\sigma = -a n_r \cos\alpha$

$\sigma = -a_T \cos\alpha \ ; \ a_T = a n_r$

$n_{max} = 0.5 n_s \Rightarrow \sigma_{min} = \dfrac{n_s - 0.5 n_s}{n_s} = 0.5$

$\sigma_{min} = -a_T \cos\alpha_{max} \Rightarrow 0.5 = -a_T \cos(170)$

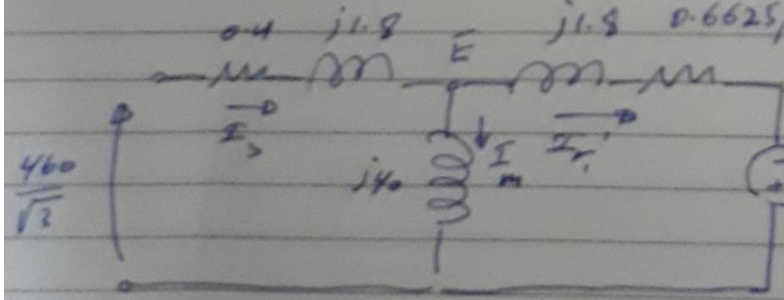
$\Rightarrow \boxed{a_T = 0.508}$

2) $\sigma = \dfrac{1200 - 720}{1200} = 0.4$

$R'_{r,new} = R'_r + R'_{f,eq} = 0.6 + a^2 R_{f,eq}$

$R'_{r,new} = 0.6 + a^2(0.5 R_f) = 0.6 + (2.5)^2(0.5 \times 0.02)$

$R'_{r,new} = 0.6625\ \Omega$



$\dfrac{a_T V \cos\alpha}{\sigma} = \dfrac{0.508}{0.4}\left(\dfrac{460}{\sqrt{3}}\right)\cos 120$

$= -168.7\ V$.

KCL at node E

$$\frac{E - 460/\sqrt{3}}{0.4 + j1.8} + \frac{E}{j40} + \frac{E - 168.7}{\left(\frac{0.6625}{0.4}\right) + j1.8} = 0$$

solve for E $\Rightarrow E = 219.3784\angle -2.6^\circ\ V$

$I'_{r} = \dfrac{E - 168.7}{\frac{0.6625}{0.4} + j1.8} = 21.023\angle -58.53^\circ\ A$.

$$T_{at} = \frac{3}{\omega_r}\left[\frac{R'_{r,new}}{\sigma} I_r'^2 + 168.7\, I_r'\right]$$

$$T_{at} = \frac{3}{1200\left(\frac{\pi}{30}\right)}\left[\frac{0.6625}{0.4}(21.02)^2 + 168.7(21.02)\right]$$

$$T_{al} = 102.144 \text{ N.m.}$$

$$\vec{I_s} = \vec{I_m} + \vec{I_r} \quad \text{or} \quad I_s = \frac{V - E}{0.4 + j1.8}$$

$$\vec{I_s} = \frac{460/\sqrt{3} - 219.3784\angle -2.6}{0.4 + j1.8}$$

$$\vec{I_s} = 25.75\angle -65.38^\circ$$

$$PF = \cos(\theta_{v_s} - \theta_{i_s}) = \cos(65.38) = 0.4166 \text{ lagging}$$

3) From the previous problem

$$T_{el,r} = 78.5 \text{ N.m}$$

$$T_{el} = \frac{3}{\omega_r}\left[\frac{R'_{r,new}}{\sigma} I'^{2}_{r_1} + \frac{-a_T V\cos\alpha}{\sigma} I'_{r_1}\right]$$

$$T_{el} = 78.5 = \frac{3}{1200\frac{\pi}{30}} \cdot \frac{1}{0.4}\left[0.6625\, I'^{2}_{r_1} - 0.508\left(\frac{460}{\sqrt{3}}\right) I'_{r_1}\cos\alpha\right]$$

$$I'^{2}_{r_1} - 203.6\, I'_{r_1}\cos\alpha = 1985 \quad \cdots\cdots ①$$

$$T_{el} \approx \frac{3V}{\omega_r} I'_{r_1} \Rightarrow 78.5 = \frac{3\left(\frac{460}{\sqrt{3}}\right)}{1200\left(\frac{\pi}{30}\right)} I'_{r_1} \quad \cdots ②$$

solving ① + ②

$$I'_{r_1} = \frac{78.5\left(1200\frac{\pi}{30}\right)}{\frac{3}{\sqrt{3}}(460)} = 12.3812 \text{ A.}$$

$$(12.3812)^2 - 203.6(12.3812)\cos\alpha = 1985$$

$$\Rightarrow \boxed{\alpha = 136.6^\circ}$$